

1.  $a > b > 0$ , both  $a$  and  $b$  are barely above 0.
2. Underflow would cause both to be rounded to 0.
3. Want to calculate  $\log(a + b)$  from  $\log(a)$  and  $\log(b)$ .

$$\begin{aligned}a + b &= e^{\log(a)} + e^{\log(b)} = 1 \times (e^{\log(a)} + e^{\log(b)}) \\ &= e^{\log(a) - \log(a)} (e^{\log(a)} + e^{\log(b)}) = e^{\log(a)} e^{-\log(a)} (e^{\log(a)} + e^{\log(b)}) \\ &= e^{\log(a)} (e^{\log(a) - \log(a)} + e^{\log(b) - \log(a)})\end{aligned}$$

Because of above and because log of product is sum of logs,

$$\begin{aligned}\log(a + b) &= \log(e^{\log(a)}) + \log(e^{\log(a) - \log(a)} + e^{\log(b) - \log(a)}) \\ \log(a + b) &= \log(a) + \log(e^{\log(a) - \log(a)} + e^{\log(b) - \log(a)})\end{aligned}$$

Assume  $\Pr(x^i, y_i = 0) = e^{-1000}$  and  $\Pr(x^i, y_i = 1) = e^{-11000}$ .

$m_i = \text{maximum of } e^{-1000} \text{ and } e^{-11000} = e^{-1000}$

$$\log(m_i) = -1000$$

Assume  $x_{i+1} = G$ ,  $p_{G0} = 0.4$ , and  $p_{G1} = 0.7$ .

Assume  $\rho_{00} = 0.8$  ( $\rho_{01} = ?$ ).

Assume  $\rho_{10} = 0.1$  ( $\rho_{11} = ?$ ).

$$\begin{aligned} \log(\Pr(x^{i+1}, y_{i+1})) &= \log(m_i) + \log\left(\sum_{y_i} \exp(-\log(m_i)) + \right. \\ &\quad \left. \log(\Pr(x^i, y_i)) + \log(p_{x_{i+1}y_{i+1}}\rho_{y_i y_{i+1}})\right) \end{aligned}$$

$$\log(\Pr(x^{i+1}, y_{i+1} = 0)) = \log(m_i) +$$

$$\log(\exp(-\log(m_i) + \log(\Pr(x^i, y_i = 0)) + \log(p_{G0}\rho_{00})) + \exp(-\log(m_i) + \log(\Pr(x^i, y_i = 1)) + \log(p_{G0}\rho_{10})))$$

$$= -1000 + \log(\exp(1000 - 1000 + \log(0.4 \times 0.8)) + \exp(1000 - 11000 + \log(0.4 \times 0.1)))$$

$$= -1000 + \log((0.4 \times 0.8) + (e^{-10000} \times 0.4 \times 0.1))$$